

Canonical Quantization of the tachyon field

Efimov G.V.

LTPb, JINR, 141980 Dubna, Russia

February 14, 2012

Abstract

The canonical quantization of the tachyon field is suggested. Quantization is based on the conception of stable and unstable components of the tachyon degrees of freedom.

1 Introduction

Last decade one can see a revival of interest to particles moving with faster-than-light speed with spectrum energy $\epsilon_k = \sqrt{k^2 - m^2} > 0$ (see [1, 2]). This particle is named a tachyon.

A classical tachyon arises in inflationary models based on the string theory and plays a definite role to explain the physics of black holes. However, recent experiments indicate the possibility that neutrino moves with faster-than-light speed. So we need a reasonable description of the tachyon field in classical and quantum physics. Usually, a tachyon is connected with the relativistic Klein-Gordon equation with negative square mass $m^2 < 0$ in order to guarantee faster-than-light speed. There are some suggestions how to quantize the tachyon field (see, for example, [3]).

In this paper, we formulate a new variant of quantization.

What is the quantization in the standard quantum field theory? The main and, unfortunately, unique practical aim is to construct the scattering S -matrix which describes all possible transitions from some free states to other free states of particles. For this aim, we need to quantize free particle fields and calculate so-called propagator, or causal Green functions of fields under consideration. Then using a given interaction Lagrangian one can construct the S -matrix in the form of perturbation expansion, each term of which is represented as a set of appropriate Feynman diagrams. A diagram is a product of appropriate propagators. The next problem is to prove that this construction satisfies all necessary conditions: relativistic covariance, unitarity, causality and so on. We will follow this scheme and our aim is to quantize the free tachyon field defined on an appropriate Fock space and calculate the tachyon causal Green function.

Our idea is quite simple. Any usual free scalar field is a set of oscillators with real frequencies $\omega_k^2 = k^2 + m^2$ for all real k , so that the scalar field quantization is a generalization of the standard quantization of oscillators to the system with an

infinite number of degrees of freedom. In the case with $m^2 < 0$ the tachyon field consists of two parts, one of which contains oscillators with real frequencies $\omega_k^2 = k^2 - m^2 > 0$ and the second contains "unstable" oscillators with imaginary frequencies $\omega_k^2 = -(m^2 - k^2) < 0$. Thus, the problem is how to perform the quantization of this "unstable" part.

In this paper, we will show how to quantize the "unstable" oscillators with $\omega^2 < 0$ and apply this procedure to the tachyon field.

2 Classical oscillator

First of all, let us consider the classical stable and unstable motion. We have two Lagrangians

$$L_{st} = \frac{\dot{q}^2(t)}{2} - \frac{\omega^2 q^2}{2}, \quad L_{un} = \frac{\dot{q}^2(t)}{2} + \frac{\Omega^2 q^2}{2}. \quad (1)$$

The Lagrangian L_{st} describes the stable finite motion and L_{un} corresponds to the unstable unbounded motion. The equations are

$$\ddot{q}_{st}(t) + \omega^2 q_{st}(t) = 0, \quad \ddot{q}_{un}(t) - \Omega^2 q_{un}(t) = 0. \quad (2)$$

The solutions for $q_{st}(t)$ and $q_{un}(t)$ look like

$$\begin{aligned} q_{st}(t) &= D_1 \cos(\omega t) + D_2 \sin(\omega t), \\ q_{un}(t) &= C_1 e^{-\Omega t} + C_2 e^{\Omega t}. \end{aligned} \quad (3)$$

Both solutions for $q_{st}(t)$ are bounded and one can formulate the Cauchy problem. However, in the second case for $q_{un}(t)$ we have increasing and decreasing, or damped, solutions. One can formulate the problem as follows: we look for the damped solutions only. It means that we should put $C_2 = 0$. On the other hand, it means that we have the boundary-value but not the Cauchy problem.

3 Stable quantum oscillator

Let us remind the canonical quantization of simple oscillator in quantum mechanics. The Hamiltonian and canonical commutation relation are

$$H = \frac{1}{2}(p^2 + \omega^2 q^2), \quad [q, p] = i. \quad (4)$$

Let us introduce the variables

$$a = \sqrt{\frac{\omega}{2}} \left(q + \frac{ip}{\omega} \right), \quad a^+ = \sqrt{\frac{\omega}{2}} \left(q - \frac{ip}{\omega} \right), \quad [a, a^+] = 1.$$

The Hamiltonian becomes the form

$$H = \frac{\omega}{2} \left(a^+ a + \frac{1}{2} \right).$$

The following commutation relations take place

$$[H, a] = -\omega a, \quad [H, a^+] = \omega a^+.$$

Let the function Ψ_E be a eigenfunction with eigenvalue E

$$H\Psi_E = E\Psi_E.$$

One can get

$$\begin{aligned} Ha\Psi_E &= (E - \omega)a\Psi_E, \\ Ha^+\Psi_E &= (E + \omega)a^+\Psi_E, \\ Ha^n\Psi_E &= (E - n\omega)a^n\Psi_E, \\ H(a^+)^n\Psi_E &= (E + n\omega)(a^+)^n\Psi_E. \end{aligned}$$

It means that the functions $a^n\Psi_E$ and $(a^+)^n\Psi_E$ are the eigenfunctions too. Our oscillator is a stable system. It means that states with arbitrary negative energy should not exist. In order to guarantee the stability, i.e. positiveness of energy we should introduce so-called vacuum, or the lowest state Ψ_0 which satisfies

$$(1) \quad a\Psi_0 = 0.$$

In the coordinate representation the vacuum looks like

$$\left(q + \frac{1}{\omega} \frac{d}{dq}\right) \Psi_0(q) = 0, \quad \Psi_0(q) = e^{-\frac{\omega}{2}q^2} \in \mathbf{L}^2.$$

Therefore, vacuum can be normalized

$$(2) \quad (\Psi_0^+ \Psi_0) = 1.$$

Eigenvalue of the vacuum is

$$H\Psi_0 = \omega \left[a^+a + \frac{1}{2} \right] \Psi_0 = \frac{\omega}{2} \Psi_0, \quad E_0 = \frac{\omega}{2}.$$

Thus, the other eigenfunctions are

$$\Psi_n = \frac{(a^+)^n}{n!} \Psi_0, \quad H\Psi_n = \omega \left(n + \frac{1}{2} \right) \Psi_n.$$

The time dependence of these eigenfunction is defined by

$$\Psi_n(t) = e^{-iHt} \Psi_n = e^{-i\omega(n+\frac{1}{2})t} \Psi_n.$$

The macroscopic oscillation is described by the coherence state $\Psi_f = e^{\frac{f}{\sqrt{2}}a^+} \Psi_0$ where f is the amplitude of oscillation, so that

$$\langle q(t) \rangle_f = \frac{(\Psi_f^+ q(t) \Psi_f)}{(\Psi_f^+ \Psi_f)} = \frac{f}{\sqrt{\omega}} \cos(\omega t).$$

The coordinate operator $q(t)$ depends on time as

$$q(t) = e^{iHt} q e^{-iHt} = \frac{1}{\sqrt{2\omega}} \left(a e^{-i\omega t} + a^\dagger e^{i\omega t} \right).$$

The commutator is

$$\Delta(t - t') = [q(t), q(t')] = \frac{1}{2\omega} \sin \omega(t - t')$$

and causal Green function, or propagator looks as

$$D_c(t - t') = (\Psi_0, T(q(t)q(t'))\Psi_0) = \frac{1}{2\omega} e^{-i\omega|t-t'|}.$$

For time ordered exponent we get

$$\left(\Psi_0, T \left\{ e^{\int_{t_0}^{t_1} dt q(t)J(t)} \right\} \Psi_0 \right) = e^{\frac{1}{2} \int_{t_0}^{t_1} \int_{t_0}^{t_1} dt dt' J(t) D_c(t-t') J(t')}$$

4 Unstable quantum oscillator

Now let us apply the stated above procedure of canonical quantization to unstable oscillator, the Hamiltonian of which is

$$H = \frac{1}{2}(p^2 - \Omega^2 q^2), \quad [q, p] = i. \quad (5)$$

Let us introduce the operators

$$A = \sqrt{\frac{\Omega}{2}} \left(q - \frac{p}{\Omega} \right), \quad B = \sqrt{\frac{\Omega}{2}} \left(q + \frac{p}{\Omega} \right), \quad [A, B] = i.$$

One should stress the operators A and B are hermitian.

The Hamiltonian becomes of the form

$$H = \frac{1}{2}(p^2 - \Omega^2 q^2) = \Omega \left(-BA - \frac{i}{2} \right).$$

The following commutation relations take place

$$[H, A] = i\Omega A, \quad [H, B] = -i\Omega B.$$

Let the function Φ_E be an eigenfunction with eigenvalue E

$$H\Phi_E = E\Phi_E.$$

One should remark that this equation, being the equation of the second order, has two independent solutions for any real or complex E .

Then one can get

$$\begin{aligned} HA\Phi_E &= (E + i\Omega)A\Phi_E, \\ HB\Phi_E &= (E - i\Omega)B\Phi_E, \\ HA^n\Phi_E &= (E + in\Omega)A^n\Phi_E, \\ HB^n\Phi_E &= (E - in\Omega)B^n\Phi_E. \end{aligned}$$

It means that the functions $A^n\Phi_E$ and $B^n\Phi_E$ are the eigenfunctions too with complex eigenvalues. The time dependence of these eigenfunctions is

$$\begin{aligned} e^{-iHt}A^n\Phi_E &= e^{-i(E+in\Omega)t}A^n\Phi_E = e^{-iEt}e^{+n\Omega t}A^n\Phi_E, \\ e^{-iHt}B^n\Phi_E &= e^{-i(E-in\Omega)t}B^n\Phi_E = e^{-iEt}e^{-n\Omega t}B^n\Phi_E. \end{aligned}$$

One can see that the eigenfunctions $A^n\Phi_E$ grow as e^{nt} . Naturally, it is not physical behavior, so by analogy with stability requirement we should impose the condition to exclude growing states. For this aim we introduce a state namad "pseudo-vacuum" Φ_0 which satisfies

$$A\Phi_0 = 0.$$

In the coordinate space it looks like

$$A\Phi_0 = \left(q + \frac{i}{\Omega} \frac{d}{dq}\right) \Phi_0(q) = 0, \quad \Phi_0(q) = e^{\frac{i\Omega}{2}q^2} \notin \mathbf{L}^2.$$

"Vacuum" energy is equal to

$$H\Phi_0 = \Omega \left[-BA - \frac{i}{2}\right] \Phi_0 = -\frac{i\Omega}{2}\Phi_0, \quad E_0 = -\frac{i\Omega}{2}.$$

The eigenfunctions and eigenvalues are

$$\begin{aligned} \Phi_n &= \frac{B^n}{\sqrt{n!}} \Phi_0, \\ H\Phi_n &= -iE_n\Phi_n, \quad E_n = \Omega \left(n + \frac{1}{2}\right). \end{aligned}$$

The time dependence of these states is

$$\Phi_n(t) = e^{-iHt}\Phi_n = e^{-\Omega(n+\frac{1}{2})t}\Phi_n.$$

It means that all states disappear when time increases $t \rightarrow \infty$.

Thus, the unstable hamiltonian has a pure imaginary spectrum. It means that the standard methods of functional analysis of hermitian operators in the functional space \mathbf{L}^2 are not applicable in our case. This situation is similar to problems with indefinite metric (see [4] and Appendix). We should introduce the rules how to calculate the matrix elements of operators depending on A and B . One can write

$$\begin{aligned} (\Phi_1, F(B)\Phi_2) &= \int dq \Phi_1(q) \cdot F\left(q + \frac{1}{i\Omega} \frac{d}{dq}\right) \Phi_2(q) \\ &= \int dq F\left(q - \frac{1}{i\Omega} \frac{d}{dq}\right) \Phi_1(q) \cdot \Phi_2(q) = (F(A)\Phi_1, \Phi_2). \end{aligned}$$

The macroscopic motion is described by the coherence state

$$\Phi_f = e^{-i\frac{f}{\sqrt{2}}B}\Phi_0, \quad \Phi_f(t) = e^{-iHt}\phi_f = e^{-\frac{\Omega}{2}t}e^{-i\frac{f}{\sqrt{2}}Be^{-\Omega t}}\Phi_0,$$

and for the coordinate $q = \frac{A+B}{\sqrt{2\Omega}}$ one can get

$$\begin{aligned} \langle q(t) \rangle_f &= \frac{(\Phi_f(t), q\Phi_f(t))}{(\Phi_f(t), \Phi_f(t))} = \frac{\left(e^{-i\frac{f}{\sqrt{2}}Be^{-\Omega t}}\Phi_0, \frac{A+B}{\sqrt{2\Omega}}e^{-i\frac{f}{\sqrt{2}}Be^{-\Omega t}}\Phi_0 \right)}{\left(e^{-i\frac{f}{\sqrt{2}}Be^{-\Omega t}}\Phi_0, e^{-i\frac{f}{\sqrt{2}}Be^{-\Omega t}}\Phi_0 \right)} \\ &= \frac{\left(\Phi_0, e^{-i\frac{f_1}{\sqrt{2}}Ae^{-\Omega t}} \frac{A+B}{\sqrt{2\Omega}} e^{-i\frac{f_2}{\sqrt{2}}Be^{-\Omega t}}\Phi_0 \right)}{\left(\Phi_0, e^{-i\frac{f_1}{\sqrt{2}}Ae^{-\Omega t}} e^{-i\frac{f_2}{\sqrt{2}}Be^{-\Omega t}}\Phi_0 \right)} \Big|_{f_1=f_2=f} \\ &= i \frac{e^{\Omega t}}{\sqrt{2\Omega}} \left(\frac{\partial}{\partial f_1} + \frac{\partial}{\partial f_2} \right) \ln \left(\Phi_0, e^{-i\frac{f_1}{\sqrt{2}}Ae^{-\Omega t}} \cdot e^{-i\frac{f_2}{\sqrt{2}}Be^{-\Omega t}}\Phi_0 \right) \Big|_{f_1=f_2=f}. \end{aligned}$$

The last matrix element can be calculated as

$$\begin{aligned} \left(\Phi_0, e^{-i\frac{f_1}{\sqrt{2}}Ae^{-\Omega t}} \cdot e^{-i\frac{f_2}{\sqrt{2}}Be^{-\Omega t}}\Phi_0 \right) &= \left(\Phi_0, e^{-i\frac{f_1}{\sqrt{2}}Be^{-\Omega t}} \cdot e^{-\frac{f_1 f_2}{2}} e^{-2\Omega t[A,B]} \cdot e^{-i\frac{f_2}{\sqrt{2}}Ae^{-\Omega t}}\Phi_0 \right) \\ &= e^{-i\frac{f_1 f_2}{2}} e^{-2\Omega t} (\Phi_0, \Phi_0). \end{aligned}$$

As a result the unstable oscillator decays exponentially

$$\langle q(t) \rangle_f = \frac{f}{\sqrt{2\Omega}} e^{-\Omega t}.$$

Let us write down the useful formulas. The coordinate operator is

$$q(t) = e^{iHt} q e^{-iHt} = e^{iHt} \frac{A+B}{\sqrt{2\Omega}} e^{-iHt} = \frac{1}{\sqrt{2\Omega}} (Ae^{-\Omega t} + Be^{\Omega t}).$$

The commutator is written as

$$[q(t), q(t')] = \frac{1}{2\Omega} [Ae^{-\Omega t} + Be^{\Omega t}, Ae^{-\Omega t'} + Be^{\Omega t'}] = -i \frac{\sinh \Omega(t-t')}{\Omega}$$

and the causal Green function, or propagator looks like

$$\begin{aligned} D_c(t-t') &= \langle T(q(t)q(t')) \rangle_0 = \frac{1}{2\Omega} \left\langle T \left((Ae^{-\Omega t} + Be^{\Omega t})(Ae^{-\Omega t'} + Be^{\Omega t'}) \right) \right\rangle_0 \\ &= \frac{i}{2\Omega} e^{-\Omega|t-t'|}. \end{aligned}$$

We would like to stress that the factor "i" has sense of the "norm" of the unstable state. Here we see an analogy with the "norm" of indefinite states, where this "norm" is equal to -1 .

The T -exponent is equal to

$$\left\langle T \left\{ e^{\int_{t_0}^{t_1} dt q(t)J(t)} \right\} \right\rangle_0 = e^{\frac{1}{2} \int_{t_0}^{t_1} \int_{t_0}^{t_1} dt dt' J(t) D_c(t-t') J(t')}$$

5 Tachyon field in QFT

The Lagrangian of the scalar tachyon field looks like

$$L(t) = \frac{1}{2} \int d\mathbf{x} \left[(\dot{\phi}(t, \mathbf{x}))^2 - (\nabla \phi(t, \mathbf{x}))^2 + m^2 \phi^2(t, \mathbf{x}) \right].$$

Let us introduce the canonical momenta $\pi(t, \mathbf{x}) = \frac{\delta L(t)}{\delta \dot{\phi}(t, \mathbf{x})} = \dot{\phi}(t, \mathbf{x})$ and write down the Hamiltonian

$$H = \frac{1}{2} \int d\mathbf{x} \left[(\pi(\mathbf{x}))^2 + (\nabla \phi(\mathbf{x}))^2 - m^2 \phi^2(\mathbf{x}) \right].$$

The canonical commutation relations are

$$[\phi(\mathbf{x}), \pi(\mathbf{x}')] = i\delta(\mathbf{x} - \mathbf{x}').$$

It is convenient to go to the momentum representation

$$\phi(\mathbf{x}) = \int \frac{d\mathbf{k}}{(2\pi)^{\frac{3}{2}}} \phi(\mathbf{k}) e^{i(\mathbf{k}\mathbf{x})}, \quad \pi(\mathbf{x}) = \int \frac{d\mathbf{k}}{(2\pi)^{\frac{3}{2}}} \pi(\mathbf{k}) e^{-i(\mathbf{k}\mathbf{x})}$$

with the canonical commutation relation

$$[\phi(\mathbf{k}), \pi(\mathbf{k}')] = i\delta(\mathbf{k} - \mathbf{k}').$$

The Hamiltonian looks like ($k^2 = \mathbf{k}^2$)

$$H = \frac{1}{2} \int d\mathbf{k} \left[\pi^2(\mathbf{k}) + (k^2 - m^2) \phi^2(\mathbf{k}) \right] = H_{in} + H_{st}$$

where

$$H_{st} = \frac{1}{2} \int d\mathbf{k} \theta(k^2 - m^2) \left[\pi^2(\mathbf{k}) + (k^2 - m^2) \phi^2(\mathbf{k}) \right],$$

$$H_{un} = \frac{1}{2} \int d\mathbf{k} \theta(m^2 - k^2) \left[\pi^2(\mathbf{k}) - (m^2 - k^2) \phi^2(\mathbf{k}) \right].$$

Both Hamiltonian describe a set of independent oscillators, but the hamiltonian H_{st} with $k^2 > m^2$ contains the real energies $E_k = \pm \sqrt{k^2 - m^2}$ and corresponds to standard stable picture. However, the Hamiltonian H_{un} with $k^2 < m^2$ contains the pure imaginary energies $E_k = \pm i \sqrt{m^2 - k^2}$ and corresponds to the instable picture.

The quantization of these two hamiltonian can be performed according procedure described in the previous sections..

5.1 Stable region $k^2 > m^2$

In the stable case the standard quantization procedure can be applied. Let us introduce the operators

$$a_{\mathbf{k}} = \sqrt{\frac{\omega_k}{2}} \left[\phi(\mathbf{k}) + i \frac{\pi(\mathbf{k})}{\omega_k} \right], \quad a_{\mathbf{k}}^+ = \sqrt{\frac{\omega_k}{2}} \left[\phi(\mathbf{k}) - i \frac{\pi(\mathbf{k})}{\omega_k} \right],$$

$$[a_{\mathbf{k}}, a_{\mathbf{k}'}^+] = \delta(\mathbf{k} - \mathbf{k}')$$

where $\omega_k = \sqrt{k^2 - m^2}$. One can see that

$$[H_{st}, a_{\mathbf{k}}] = -\omega_k a_{\mathbf{k}}, \quad [H_{st}, a_{\mathbf{k}}^+] = \omega_k a_{\mathbf{k}}^+.$$

The lowest state, or vacuum is defined by the condition

$$a_{\mathbf{k}} \Psi_0 = 0.$$

Then the stable part of the tachyon field has the standard form

$$\eta_{\mathbf{k}} = \frac{1}{\sqrt{2\omega_k}} (a_{\mathbf{k}} + a_{\mathbf{k}}^+),$$

$$\eta_{\mathbf{k}}(t) = e^{iH_{st}t} \phi_{\mathbf{k}} e^{-iH_{st}t} = \frac{1}{\sqrt{2\omega_k}} (a_{\mathbf{k}} e^{-i\omega_k t} + a_{\mathbf{k}}^+ e^{i\omega_k t}),$$

$$\eta(t, \mathbf{x}) = \int \frac{d\mathbf{k}}{(2\pi)^{\frac{3}{2}}} \frac{\theta(k^2 - m^2)}{\sqrt{2\omega_k}} (a_{\mathbf{k}} e^{-i\omega_k t + i\mathbf{k}\mathbf{x}} + a_{\mathbf{k}}^+ e^{i\omega_k t - i\mathbf{k}\mathbf{x}})$$

5.2 Unstable region $k^2 < m^2$

In the unstable case the quantization procedure developed for unstable oscillator can be applied. Let us introduce the operators

$$A_{\mathbf{k}} = \sqrt{\frac{\Omega_k}{2}} \left[\phi(\mathbf{k}) - \frac{\pi(\mathbf{k})}{\Omega_k} \right], \quad B_{\mathbf{k}} = \sqrt{\frac{\Omega_k}{2}} \left[\phi(\mathbf{k}) + \frac{\pi(\mathbf{k})}{\Omega_k} \right],$$

where $\Omega_k = \sqrt{m^2 - k^2}$. The commutation relations are

$$[A_{\mathbf{k}}, B_{\mathbf{k}'}] = i\delta(\mathbf{k} - \mathbf{k}').$$

These commutation relations lead to

$$[H_{in}, A_{\mathbf{k}}] = i\Omega_k A_{\mathbf{k}}, \quad [H_{in}, B_{\mathbf{k}}] = -i\Omega_k B_{\mathbf{k}}.$$

Let the function Φ_E be an eigenfunction with eigenvalue E

$$H\Phi_E = E\Phi_E.$$

Then one can get

$$HA_{\mathbf{k}}\Phi_E = (E + i\Omega_k)A_{\mathbf{k}}\Phi_E,$$

$$HB_{\mathbf{k}}\Phi_E = (E - i\Omega_k)B_{\mathbf{k}}\Phi_E.$$

It means that the functions $A_{\mathbf{k}}\Phi_E$ and $B_{\mathbf{k}}\Phi_E$ are the eigenfunctions too with complex eigenvalues. The time dependence of these eigenfunctions is

$$\begin{aligned} e^{-iHt} A_{\mathbf{k}}\Phi_E &= e^{-iEt+\Omega_k t} A_{\mathbf{k}}\Phi_E, \\ e^{-iHt} B_{\mathbf{k}}\Phi_E &= e^{-iEt-\Omega_k t} A_{\mathbf{k}}\Phi_E. \end{aligned}$$

One can see that the eigenfunctions $A_{\mathbf{k}}\Phi_E$ grow as $e^{\Omega t}$. From a standard point of view it is not physical behavior, so by analogy with the stability requirement we should impose the condition, namely, we introduce a state - pseudo-vacuum Φ_0 which satisfies

$$A_{\mathbf{k}}\Phi_0 = 0.$$

All further argumentations and calculations repeat literally the results of the section 4. As a result we have for the instable part of the tachyon field

$$\begin{aligned} \chi_{\mathbf{k}} &= \frac{1}{\sqrt{2\Omega_k}}(A_{\mathbf{k}} + B_{\mathbf{k}}), \\ \chi_{\mathbf{k}}(t) &= e^{iH_{in}t} \phi_{\mathbf{k}} e^{-iH_{in}t} = \frac{1}{\sqrt{2\Omega_k}} \left(A_{\mathbf{k}} e^{-\Omega_k t} + B_{\mathbf{k}} e^{\Omega_k t} \right), \\ \chi(t, \mathbf{x}) &= \int \frac{d\mathbf{k}}{(2\pi)^{\frac{3}{2}}} \frac{\theta(m^2 - k^2)}{\sqrt{2\Omega_k}} \left(A_{\mathbf{k}} e^{-\Omega_k t + i\mathbf{k}\mathbf{x}} + B_{\mathbf{k}} e^{\Omega_k t - i\mathbf{k}\mathbf{x}} \right). \end{aligned}$$

5.3 The tachyon field

The tachyon field is a sum of stable and unstable fields

$$\phi(t, \mathbf{x}) = \eta(t, \mathbf{x}) + \chi(t, \mathbf{x})$$

for which the vacuum is

$$|0\rangle = \Psi_0 \Phi_0.$$

One tachyon states are

$$\begin{aligned} \langle 0 | \phi(t, \mathbf{x}) a_{\mathbf{k}}^+ | 0 \rangle &= \frac{1}{(2\pi)^{\frac{3}{2}} \sqrt{2\omega_k}} e^{-i\omega_k t + i\mathbf{k}\mathbf{x}} \\ \langle 0 | \phi(t, \mathbf{x}) B_{\mathbf{k}} | 0 \rangle &= \frac{i}{(2\pi)^{\frac{3}{2}} \sqrt{2\Omega_k}} e^{-\Omega_k t + i\mathbf{k}\mathbf{x}} \end{aligned}$$

One can see that the unstable states decrease exponentially.

The causal Green function, or propagator looks like

$$\begin{aligned} D_c(t - t', \mathbf{x} - \mathbf{x}') &= \langle T(\phi(t, \mathbf{x}) \phi(t', \mathbf{x}')) \rangle_0 \\ &= \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{\theta(k^2 - m^2)}{2\sqrt{k^2 - m^2}} e^{-i\omega_k |t - t'| + i\mathbf{k}(\mathbf{x} - \mathbf{x}')} + i \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{\theta(m^2 - k^2)}{2\sqrt{m^2 - k^2}} e^{-\Omega_k |t - t'| + i\mathbf{k}(\mathbf{x} - \mathbf{x}')} \end{aligned}$$

This representation can be represented in the usual form

$$\begin{aligned}
D_c(t, \mathbf{x}) &= \int_m^\infty \frac{dk \, k^2}{(2\pi)^2 2\sqrt{k^2 - m^2}} e^{-i\omega_k|t-t'|} \frac{\sin(kx)}{xk} + i \int_0^m \frac{dk \, k^2}{(2\pi)^2 2\sqrt{m^2 - k^2}} e^{-\Omega_k|t-t'|} \frac{\sin(kx)}{xk} \\
&= \int_C \frac{dk \, k^2}{(2\pi)^2 2\sqrt{k^2 - m^2}} e^{-i\omega_k|t-t'|} \frac{\sin(kx)}{xk} \\
&= \int \frac{d^4q}{(2\pi)^4 i} \cdot \frac{e^{-iqx}}{(q^2 + m^2 + i0)}, \quad q^2 = q_0^2 - \mathbf{q}^2.
\end{aligned}$$

6 Conclusion

The asymptotic tachyon field contains only the stable field

$$\phi_{in}(t, \mathbf{x}) = \phi_{out}(t, \mathbf{x}) = \eta(t, \mathbf{x})$$

for which the vacuum is the standard Fock vacuum $|0\rangle = \Psi_0$ with one particle state

$$\langle 0 | \eta(t, \mathbf{x}) a_{\mathbf{k}}^+ | 0 \rangle = \frac{1}{(2\pi)^{\frac{3}{2}} \sqrt{2\omega_k}} e^{-i\omega_k t + i\mathbf{k}\mathbf{x}}, \quad \omega_k^2 = k^2 - m^2 > 0.$$

It means that the unstable tachyon components do not exist as physical states.

The formula

$$\langle 0 | T \left\{ e^{\int dx \, \phi(x) J(x)} \right\} | 0 \rangle = e^{\frac{1}{2} \iint dx dx' \, J(x) D_c(x-x') J(x')}$$

with

$$D_c(t, \mathbf{x}) = \int \frac{d^4q}{(2\pi)^4 i} \cdot \frac{e^{-iqx}}{(q^2 + m^2 + i0)}, \quad q^2 = q_0^2 - \mathbf{q}^2$$

permits one to construct the S -matrix in the form of the standard perturbation decomposition in the form of Feynman diagrams.

Our scheme is similar to quantization of the electro-magnetic field when the photon field has two physical (transverse) and two nonphysical (longitudinal and time) components. The nonphysical components do not exist as physical states, the norm of these states is not a positive number, but they are needed to construct the photon causal propagator.

The next step is to introduce an interaction of the tachyon with other particles, calculate amplitudes for different processes and analyze these amplitudes of perturbation expansion, but it is a different story with numerous problems.

7 Appendix

Let us introduce the linear space with complex metric \mathcal{H}_c . A vector $|\Phi\rangle$ belongs to \mathcal{H}_c , if

1. The scalar product (Φ_1, Φ_2) is defined;

2. The scalar product (Φ, Φ) is a complex number;
3. There exist two operators A and B which satisfy the conditions
 - if $\Phi \in \mathcal{H}_c$, then $A\Phi \in \mathcal{H}_c$ and $B\Phi \in \mathcal{H}_c$;
 - commutation relations are

$$[A, B] = i; \quad (6)$$

- the following equalities take place:

$$(\Phi_1, B\Phi_2) = (A\Phi_1, \Phi_2), \quad (\Phi_1, A\Phi_2) = (B\Phi_1, \Phi_2); \quad (7)$$

4. There exists the vector, named "pseudo-vacuum", $\Phi_0 \in \mathcal{H}_c$, which satisfies

$$(1) \quad A\Phi_0 = 0, \quad (2) \quad (\Phi_0, \Phi_0) \neq 0. \quad (8)$$

The vector space \mathcal{H}_c occupies an intermediate place between the standard Hilbert space and the vector space with indefinite metric (see [4]).

If the conditions (6),(7) and (8) take place, then one can get

$$\begin{aligned} (e^{f_1 B} \Phi_0, e^{f_2 B} \Phi_0) &= (\Phi_0, e^{f_1 A} \cdot e^{f_2 B} \Phi_0) = (\Phi_0, e^{f_2 B} \cdot e^{f_1 f_2 [A, B]} \cdot e^{f_1 A} \Phi_0) \\ &= (e^{f_2 A} \Phi_0, e^{i f_1 f_2} \cdot e^{f_1 A} \Phi_0) = e^{i f_1 f_2} (\Phi_0, \Phi_0) \end{aligned} \quad (9)$$

and

$$M_{f_1, f_2} = \frac{(e^{f_1 B} \Phi_0, e^{f_2 B} \Phi_0)}{(\Phi_0, \Phi_0)} = e^{i f_1 f_2}. \quad (10)$$

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